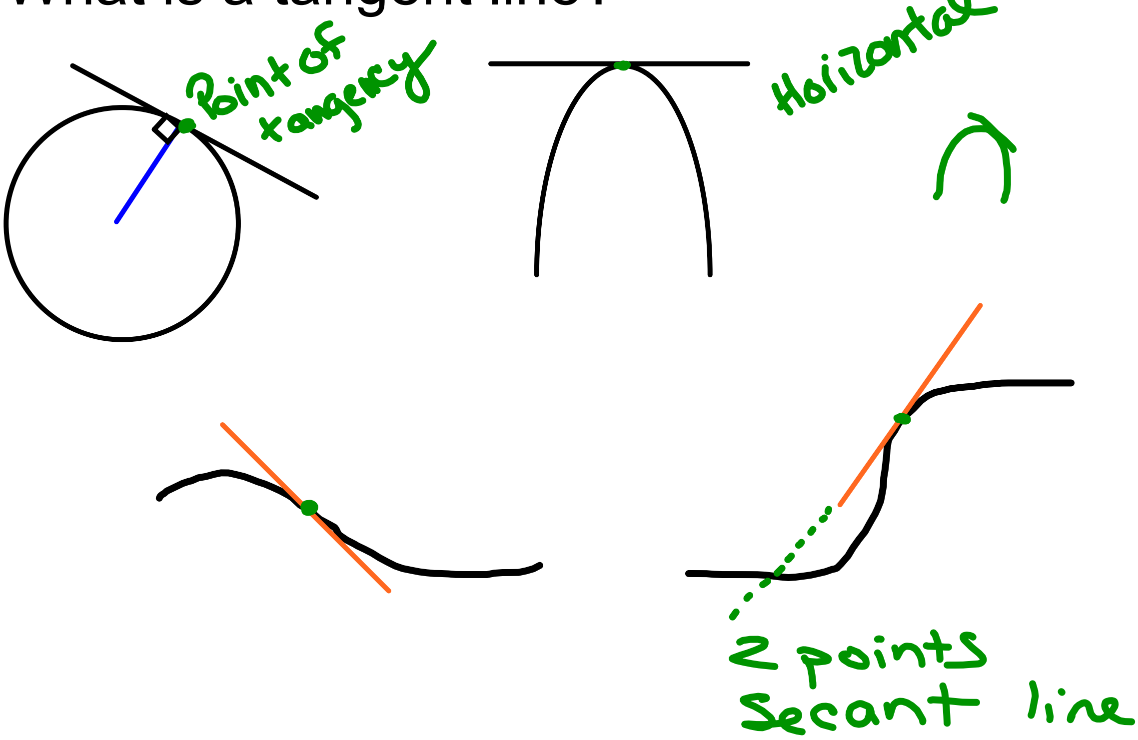
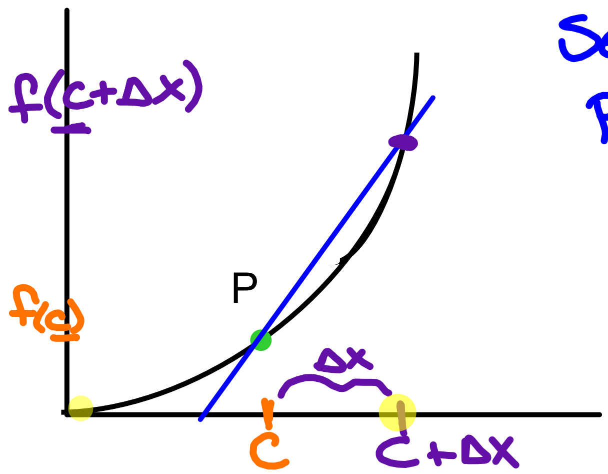
The Derivative and the Tangent Line

What is a tangent line?





Secant line through a point of tangency & another point

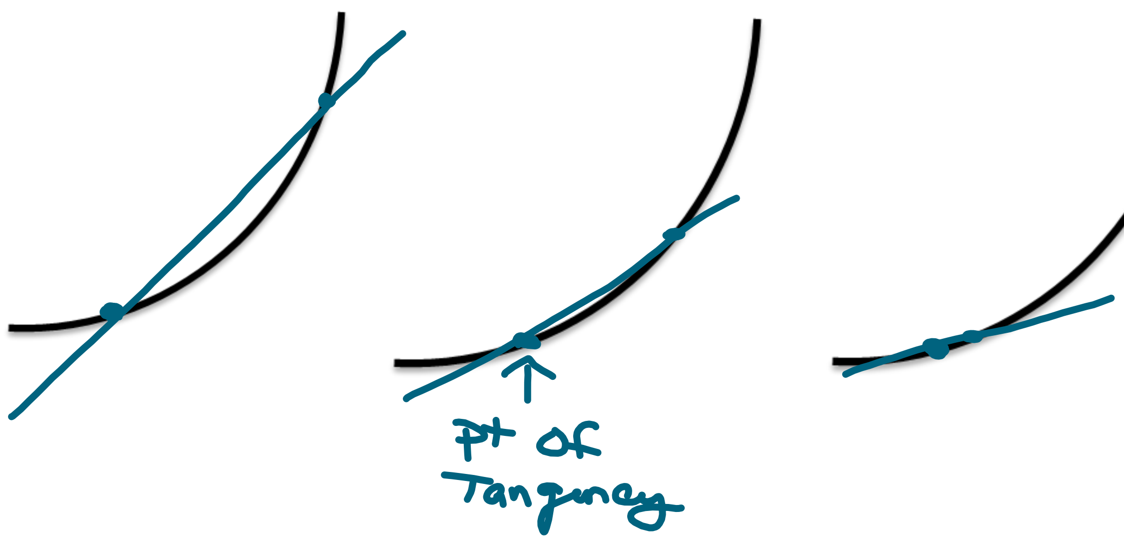
find slope of secant line

(avg rate of change)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(c+\Delta x) - f(c)}{\cancel{c} + \Delta x - \cancel{c}}$$

$$m_{sec} = \frac{f(c+\Delta x) - f(c)}{\Delta x} \qquad \frac{\Delta y}{\Delta x}$$

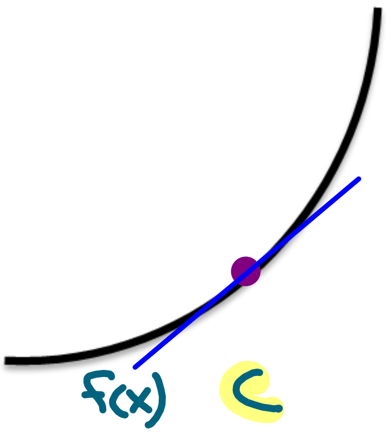
Difference Quotient


pt of
Tangency

Definition of Tangent line with slope m

$$\lim_{x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

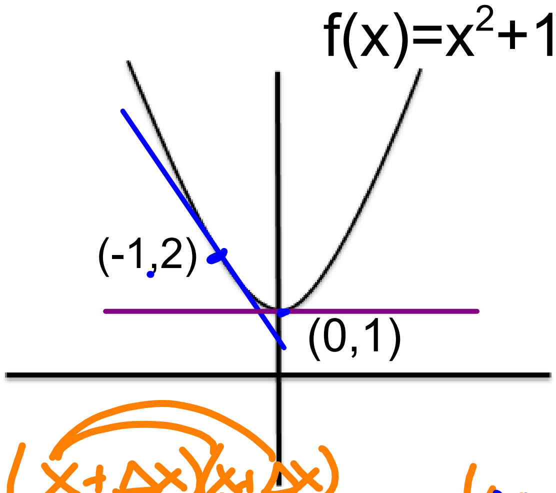
If the limit exists, then the line passing through the point of tangency (c, f(c)) with slope m is the tangent line.



the slope of the tangent line at point c is also

the slope of the graph f(x) at x=c

Instaneous rate of change



Find the slope at the given points.

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$(x+\Delta x)(x+\Delta x)$

$x^2 + x\Delta x + x\Delta x + \Delta x^2$

$$\lim_{\Delta x \to 0} \frac{((x+\Delta x)^2+1) - (x^2+1)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 + \cancel{1} - \cancel{x^2} - \cancel{1}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x} = \frac{\cancel{\Delta x}(2x+\Delta x)}{\cancel{\Delta x}}$$

$$\boxed{\lim_{\Delta x \to 0} 2x+\Delta x}$$

$2x$

(-1,2)

$2(-1)$

$\boxed{m=-2}$

$2x$

(0,1)

$2(0)$

$\boxed{m=0}$

Horizontal Tangent line

$f(x) = x^2 + 1$

$f(2) = 2^2 + 1$

$f(2) = \underline{\underline{5}}$

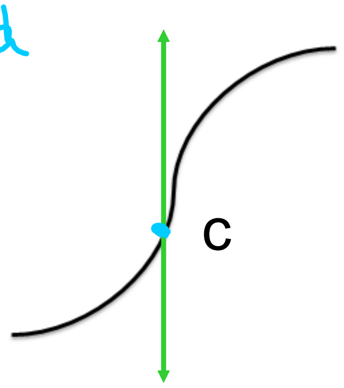
y

Vertical Tangent Line - If $f(x)$ is continuous at c. and

$$\lim_{\Delta x \to 0} \frac{f(c+\Delta x)-f(c)}{\Delta x} = \infty$$

or

$$\lim_{\Delta x \to 0} \frac{f(c+\Delta x)-f(c)}{\Delta x} = -\infty$$



$\therefore$ there is a vertical Tangent line at c

Derivative

Used to determine instantaneous rate of change

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

notation for the derivative

The limit must exist.

Notation for derivatives:

$f'(x)$, $\boxed{\frac{dy}{dx}}$, y', $\frac{d}{dx}[f(x)]$, $D_x[y]$

the derivative of y
w/ respect to x

The process of finding the derivative is called differentiation.

$f'(x)$ = 1st deriv.
$f''(x)$ = 2nd deriv.

a function is differentiable if
* the derivative exists
* the function is differentiable at every point on (a, b)

Find the derivative of

$$f(x) = x^3 + 2x$$

$$f(x+\Delta x) = (x+\Delta x)^3 + 2(x+\Delta x)$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 + 2(x+\Delta x) - (x^3+2x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{x^3} + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + \cancel{2x} + 2\Delta x - \cancel{x^3} - \cancel{2x}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(3x^2 + 3x\Delta x + (\Delta x)^2 + 2)}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} 3x^2 + 3x\Delta x + (\Delta x)^2 + 2$$

$$3x^2 + 3x(0) + 0^2 + 2$$

$$\boxed{f'(x) = 3x^2 + 2}$$

Find $f'(x)$ for $f(x) = \sqrt{x}$

Then find the slopes of the graph at (1,1) and (4,2)

$$\lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{(\sqrt{x+\Delta x} + \sqrt{x})}{(\sqrt{x+\Delta x} + \sqrt{x})}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{x} + \cancel{\Delta x} - \cancel{x}}{\cancel{\Delta x}(\sqrt{x+\Delta x} + \sqrt{x})}$$

$$\lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$\frac{1}{\sqrt{x} + \sqrt{x}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

3, 5-19 odd, 23

(1,1)

$$\frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$m = \frac{1}{2}$

(4,2)

$$\frac{1}{2\sqrt{4}} = \frac{1}{4}$$